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MHD FLOW OVER AN EXPONENTIALLY STRETCHING SHEET WITH VISCOUS DISSIPATION AND HEAT GENERATION

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I. INTRODUCTION

In the last few decades, fluid flow with heat and mass transfer on a continuously stretching sheet has attracted considerable attention because of its many applications in industries, engineering and manufacturing processes. Examples of these applications include the glass-fiber production, wire drawing, paper production, plastic sheets, metal and polymer processing industries, hot rolling and continuous casting of metals and spinning of fibers. The kinematics of stretching and the simultaneous heating or cooling during such processes play an important role on the structure and quality of the final product. Many researchers inspired by Sakiadis [1,2] who initiated the boundary layer behavior studied the stretching flow problem in various aspects. Extension to that, an exact solution was given by Crane [3] for a boundary layer flow caused by stretching surface. Gupta [4], Carragher and Crane [5], Dutta *et al.* [6] studied the heat transfer in the flow over a stretching surface taking into account different aspects of the problem. Magyari and Keller [7] observed that the study of boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Sanjayanand and Khan [8] studied the heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet. They found that the viscoelastic parameter enhances the thermal boundary layer thickness. The effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface was studied by Partha *et al.* [9]. They observed a rapid growth in the non-dimensional skin friction coefficient with the mixed convection parameter.

The characteristics desired of the final product in an extrusion process depend on the rate of stretching and cooling. Hence, it is very important to have a controlled cooling environment where the flow over the stretching sheet can be regulated by external agencies like a magnetic field. An exponential variation of a magnetic field is used, among other applications, to determine the diamagnetic susceptibility of plasma. Pavlov [10] considered the magnetohydrodynamic flow of an incompressible viscous fluid over a linearly stretching surface. Sarpakaya [11] extended Pavlov's work to non-Newtonian fluids. Subsequent studies by Andersson [12], Lawrence and Rao [13], Abel *et al.* [14], Cortell [15] concerned the magnetohydrodynamic flow of viscoelastic liquids over a stretching sheet.

Most of the earlier work neglected radiation effects. If the polymer extrusion process is placed in a thermally controlled environment, radiation could become important. Many researchers have considered the effect of thermal radiation on flows over stretching sheets. The influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet is studied by Sajid and Hayat [16]. Studies by Raptis [17], Raptis and Perdikis [18] address the effect of radiation in various situations. The effects of radiation on hydromagnetic boundary layer flow of a continuously stretching surface have attracted considerable attention in recent times due to its numerous applications in industry. Kameswaran *et al.* [19] observed that radiation effects on MHD Newtonian liquid flow due to an exponential stretching sheet. Seini and Makinde [20] studied effects of radiation and chemical reaction on MHD boundary layer flow over exponential stretching surface. Siddheshwar and Mahabaleswar [21] studied the effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Bidin and Nazar [22] studied the effects of numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. Elbashbeshy and Dimian [23] analyzed boundary layer flow in the presence of radiation effect and heat transfer over the wedge with a viscous coefficient. Thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet were studied by Reddy and Reddy [24]. Raptis *et al.* [25] studied the effect of thermal radiation on the magnetohydrodynamic flow of a viscous fluid past semi-infinite stationary plate and Hayat *et al.* [26] extended the analysis for the second grade fluid. Jat and Gopi Chand [27] found that the effects of dissipation and radiation on MHD flow and heat transfer over an exponentially stretching

sheet. Radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of a heat source was studied by Reddy *et al.* [28]. They observed that the velocity decreases with an increase in the magnetic parameter due to a resistive drag force which tends to resist the fluid flow and thus reduces the velocity. The boundary layer thickness was also found to decrease with an increase in the magnetic parameter.

In addition to a magnetic field and thermal radiation, one has to consider the viscous dissipation effects due to frictional heating between fluid layers. The effect of viscous dissipation in natural convection processes has been studied by Gebhart [29] and Gebhart and Mollendorf [30]. They observed that the effect of viscous dissipation is predominant in vigorous natural convection and mixed convection processes. They also showed the existence of a similarity solution for the external flow over an infinite vertical surface with an exponential variation of surface temperature.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution and this in turn can affect the particle deposition rate in nuclear reactors, electronic chips and semi conductor wafers. Although exact modeling of internal heat generation or absorption on is quite difficult, some simple mathematical models can be used to express its general behavior for most physical situations. Vajravelu and Hadjinicalaou [31] studied the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Mohammed Ibrahim and Bhaskar Reddy [32] noticed that effects of radiation and mass transfer effects on MHD flow along a stretching surface in presence of viscous dissipation and heat generation. Pavithra and Gireesha [33] noticed that the effect of heat generation on dusty fluid flow over an exponentially stretching sheet with viscous dissipation. Mohammed Ibrahim [34] found that radiation effects on mass transfer flow passing through a highly porous medium in presence of heat generation and chemical reaction. Mohammed Ibrahim [35] investigated effects of chemical reaction and radiation on MHD free convection flow along a stretching surface in presence of dissipation and heat generation.

We investigate the effects of various physical and fluid parameters such as the magnetic parameter, radiation parameter, viscous dissipation parameter, heat generation parameter and chemical reaction parameter on the flow, heat and mass transfer characteristics of an exponentially stretching sheet. The momentum, energy and concentration equations are coupled and nonlinear. By using suitable similarity variables, these equations are coupled and nonlinear. By using suitable similarity variable, these equations are converted into coupled ordinary differential equations and are solved numerically by using shooting technique with the forth order Runge-Kutta method.

II. FORMULATION OF THE PROBLEM

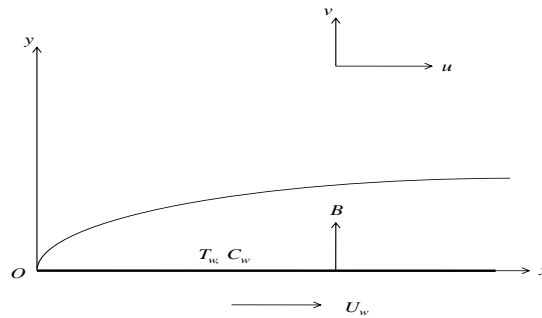


Fig. 1 Schematics of the problem

Consider a steady two dimensional laminar flow of a viscous incompressible electrically conducting fluid over a continuous exponentially stretching surface. The x – axis is taken along the stretching surface in the direction of motion and y - axis is perpendicular to it. The sheet velocity is assumed to vary as an exponential function of the distance x from the slit. The temperature and concentration far away from the fluid are assumed to be T_∞ and C_∞ respectively as shown in Figure 1. The sheet-ambient temperature and concentration differences are also assumed to be exponential functions of the distance x from the slit. A variable magnetic field of strength $B(x)$ is applied normally to the sheet. Under the usual boundary layer approximation, subject to radiation, viscous dissipation, heat generation and chemical reaction effects, the equations governing the momentum, heat and mass transports can be written as Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{Q_0}{\rho c_p} (T - T_\infty) \tag{3}$$

Species equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Kr^* (C - C_\infty) \tag{4}$$

where u and v are the velocity components in the x, y directions respectively, ν is the kinematic viscosity, ρ is the density, σ is the electrical conductivity of the fluid, T is the temperature, C is the concentration, k is the thermal conductivity, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, Q_0 is the heat generation coefficient, D is the species diffusivity, Kr^* is the reaction rate parameter.

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned}
 u = U_w = U_0 e^{\frac{x}{L}}, \quad v = 0, \quad T = T_w = T_\infty + T_0 e^{\frac{2x}{L}}, \quad C = C_w = C_\infty + C_0 e^{\frac{2x}{L}} \quad \text{at } y = 0 \\
 u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty
 \end{aligned} \tag{5}$$

Here the subscripts w, ∞ refer to the surface and ambient conditions respectively, T_0, C_0 are positive constants, U_0 is the characteristic velocity, L is the characteristic length and Q is the constant.

To facilitate a similarity solution, the magnetic field $B(x)$ is assumed to be of the form

$$B(x) = B_0 e^{\frac{x}{2L}} \tag{6}$$

where B_0 is a constant. It is also assumed that the fluid is weakly electrically conducting so that the induced magnetic field is negligible. Following Rosseland’s approximation, the radiative heat flux q_r is modeled as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{7}$$

where σ^* is the Stefan-Boltzman constant, k^* is the mean absorption coefficient.

This approximation is valid at points optically far from the boundary surface and it is good for intensive absorption, which is for an optically thick boundary layer. It is assumed that the temperature difference within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 by Taylor series about T_∞ and neglecting higher-order terms gives:

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4$$

$$\text{We have } \frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

Continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

The following similarity variables are used:

$$\begin{aligned}
 u = U_0 e^{\frac{x}{L}} f'(\eta), \quad v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} (f(\eta) + \eta f'(\eta)) \\
 T = T_\infty + T_0 e^{\frac{2x}{L}} \theta(\eta), \quad C = C_\infty + C_0 e^{\frac{2x}{L}} \phi(\eta), \quad \eta = \sqrt{\frac{U_0}{2\nu L}} y e^{\frac{x}{2L}}
 \end{aligned} \tag{10}$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, and $\phi(\eta)$ is the dimensionless concentration.

On using equations (6), (8) and (10), equations (2) – (5) are transformed to:

$$f''' - 2f'^2 + ff'' - Mf' = 0 \tag{11}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \text{Pr} [f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta] = 0 \tag{12}$$

$$\phi'' + Scf\phi' - Scf'\phi - ScKr\phi = 0 \tag{13}$$

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) \rightarrow 0, \quad (14)$$

$$\theta(0) = 1, \quad \theta(\infty) \rightarrow 0, \quad (15)$$

$$\phi(0) = 1, \quad \phi(\infty) \rightarrow 0. \quad (16)$$

The non-dimensional constants appearing in equations (11) – (13) are the magnetic parameter M , the radiation parameter R , the Prandtl number Pr , the Eckert number Ec , Q is the heat generation parameter, Sc is the Schmidt number and Kr is the chemical reaction parameter respectively defined as

$$M = \frac{2\sigma B_0^2 L}{\rho U_0} \quad R = \frac{4\sigma^* T_\infty^3}{k^* k}, \quad Pr = \frac{\rho \nu c_p}{k} \quad Ec = \frac{U_0^2}{c_p T_0}$$

$$Q = \frac{2Q_0 L}{U_0 e^{\frac{x}{L}}}, \quad Sc = \frac{\nu}{D}, \quad Kr = \frac{2LKr^*}{U_0}$$

Skin friction, heat and mass transfer coefficients

The parameters of engineering interest in heat and mass transport problems are the skin friction- coefficient C_f , the local Nusselt number Nu , and the local Sherwood number Sh . These parameters respectively characterize the surface drag, wall heat and mass transfer rates.

The shearing stress at the surface of the wall τ_w is given by

$$\tau_w = -\mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = -\frac{\mu U_0}{L} \sqrt{\frac{Re}{2}} e^{\frac{3x}{2L}} f''(0), \quad (17)$$

where μ is the coefficient of viscosity and $Re = \frac{U_0 L}{\nu}$ is the Reynolds number. The skin friction coefficient is defined as

$$C_f = \frac{2\tau_w}{\rho U_w^2} \quad (18)$$

and using equation (17) in equation (18), we obtain

$$\frac{C_f \sqrt{\frac{Re}{2}}}{\sqrt{\frac{x}{L}}} = -f''(0). \quad (19)$$

The heat transfer rate at the surface flux at the wall is given by

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = \frac{-k(T_w - T_\infty)}{L} \sqrt{\frac{Re}{2}} e^{\frac{x}{2L}} \theta'(0), \quad (20)$$

where k is the thermal conductivity of the fluid. The Nusselt number is defined as

$$Nu = \frac{x}{k} \frac{q_w}{T_w - T_\infty}. \quad (21)$$

Using Equation (20) in Equation (21), the dimensionless wall heat transfer rate is obtained as follows:

$$\frac{Nu}{\sqrt{\frac{x}{L}} \sqrt{\frac{Re}{2}}} = -\theta'(0). \quad (22)$$

The mass flux at the surface of the wall is given by

$$J_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} = \frac{-D(C_w - C_\infty)}{L} \sqrt{\frac{Re}{2}} e^{\frac{x}{2L}} \phi'(0) \quad (23)$$

The Sherwood number is defined as

$$Sh = \frac{x}{D} \frac{J_w}{C_w - C_\infty} \quad (24)$$

Using (23) in (24), the dimensionless wall mass transfer rate is obtained as

$$\frac{Sh}{\sqrt{\frac{x}{L}} \sqrt{\frac{Re}{2}}} = -\phi'(0) \quad (25)$$

In Equations (19), (22) and (25), Re represents the local Reynolds number and it is defined as

$$Re = \frac{xU_w}{\nu}$$

III. NUMERICAL PROCEDURE

The set of nonlinear ordinary differential equations (11), (12), and (13) with boundary conditions (14) - (16) were solved numerically using Runge – Kutta fourth order algorithm with a systematic guessing of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution and accuracy up to the fifth decimal place i.e. 1×10^{-5} , which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta \rightarrow \infty$, say η_∞ , which depend on the values of the parameter used. The computations were done by a program which uses a symbolic and computational computer language in Mathematica.

IV. RESULTS AND DISCUSSION

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various in governing parameters, namely, magnetic field parameter M , Prandtl number Pr , radiation parameter R , heat generation parameter Q , Eckert number Ec , Schmidt number Sc , chemical reaction parameter Kr . In the present study following default parameter values are adopted for computations: $M = 1.0$, $Pr = 0.71$, $R = 0.5$, $Q = 0.1$, $Ec = 0.1$, $Sc = 0.6$, $Kr = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figure 2 shows the variation of the velocity profile against the magnetic parameter. We notice that the effect of the magnetic parameter is to reduce the velocity of the fluid in the boundary layer region. This is due to an increase in the Lorentz force, similar to Darcy’s drag observed in the case of flow through a porous medium. This adverse force is responsible for slowing down the motion of the fluid in the boundary layer region.

The variation of the temperature distribution with the magnetic parameter is shown in Figure 3. The thermal boundary layer thickness increases with increasing values of the magnetic parameter. The opposing force introduced in the form of the Lorentz drag contributes in increasing the frictional heating between the fluid layers, and hence energy is released in the form of heat. This results in thickening of the thermal boundary layer.

The effect of the magnetic parameter on the concentration profile is shown in Figure 4. It is observed that increases in the values in M result in thickening of the species boundary layer.

Fig. 5 illustrates the temperature profile for different values of the Prandtl number Pr . The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show the effect of increasing values

of Prandtl number results in decreasing temperature. The reason is that smaller values of Prandtl number Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Prandtl number Pr . Hence in the case of smaller Prandtl number Pr as the boundary layer is thicker and the rate of heat transfer is reduced.

The influence of the thermal radiation parameter R on temperature is shown in Figure 6. It is clear that thermal radiation enhances the temperature in the boundary layer region. Thus radiation should be kept at its minimum in order to facilitate better cooling environment. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer.

The effect of the Eckert number Ec on heat transfer is shown in Figure 7. It is clear that the temperature in the boundary layer region increases with an increase in the viscous dissipation parameter.

Figure 8 shows the influence of the heat generation parameter Q on the temperature profile within the thermal boundary layer. From the Figure 8 it is observed that the temperature increases with an increase in the heat generation parameter.

Figures 9-10 depict chemical species concentration profiles against co-ordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the concentration boundary layer thickness decreases with an increase in Schmidt number and chemical reaction parameter.

We also note that since the energy equation is partially decoupled from the momentum and species conservation equations, the parameters affecting the energy equation namely the Prandtl number, the thermal radiation parameter, heat generation parameter and the Eckert number, do not alter velocity and concentration profiles.

Table 1 shows the comparison of Kameswaran *et al.* [19] work with the present work for $Pr = R = Ec = Q = Sc = Kr = 0$ and it note worthy that there is a good agreement.

Table 2 indicates the values of skin-friction coefficient, the wall temperature gradient and the wall concentration gradient in terms of $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ respectively for various values embedded flow parameter. From Table 2, it is understood that, as increasing values of magnetic field parameter (M) results in considerable opposition to the flow in the form of a Lorenz drag which enhances the values of skin-friction coefficient, but there is a decrease in the wall temperature gradient and the wall concentration gradient. The wall temperature gradient reduces as increase the values of radiation parameter R or dissipation Ec or heat generation parameter Q , while it is increases for increasing value of Prandtl number Pr . It is also observed that the increase in Schmidt number Sc or chemical reaction parameter Kr parameter lead to the increase in the dimensionless wall concentration gradient.

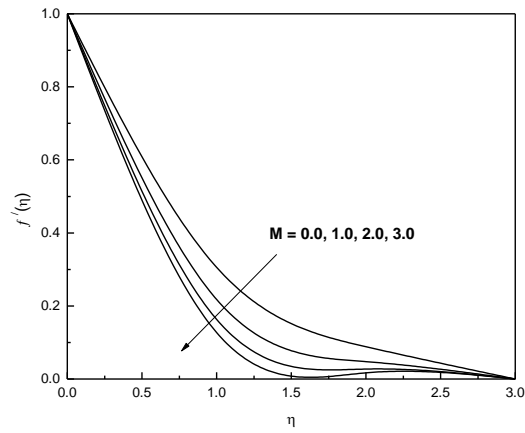


Fig. 2. Velocity profiles for varying values of magnetic parameter (M)

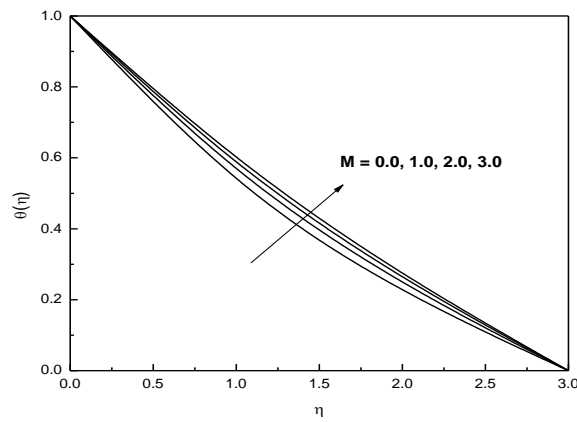


Fig. 3. Temperature profiles for varying values of magnetic parameter (M)

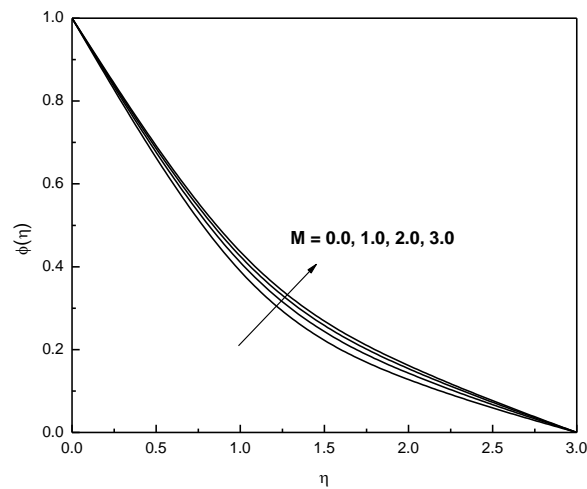


Fig. 4. Concentration profiles for varying values of magnetic parameter (M)

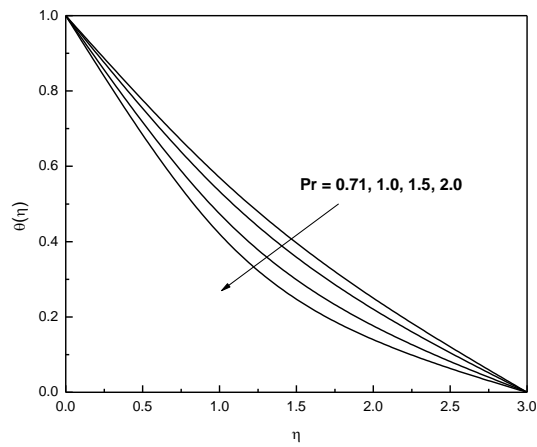


Fig. 5. Temperature profiles for varying values of Prandtl parameter (*Pr*)

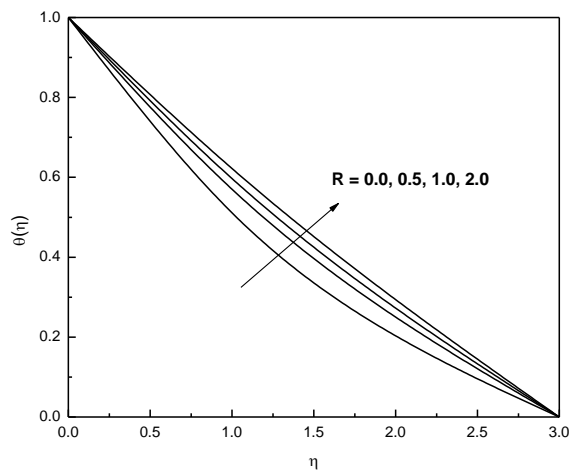


Fig. 6. Temperature profiles for varying values of radiation parameter (*R*)

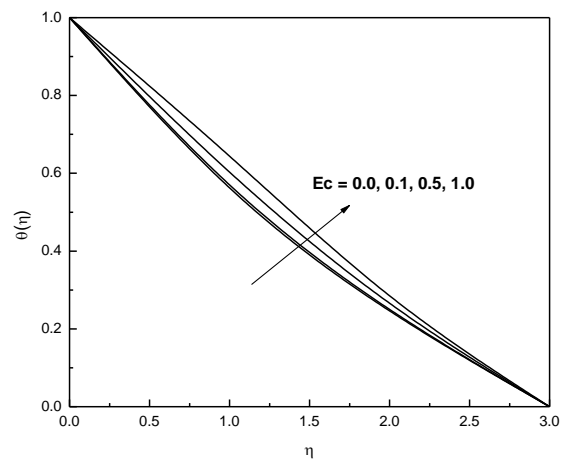


Fig. 7. Temperature profiles for varying values of viscous dissipation parameter (Ec)

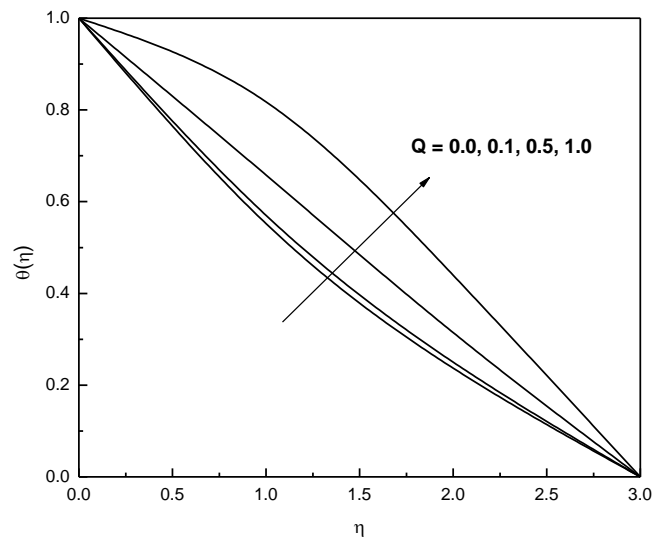


Fig. 8. Temperature profiles for varying values of heat generation parameter (Q)

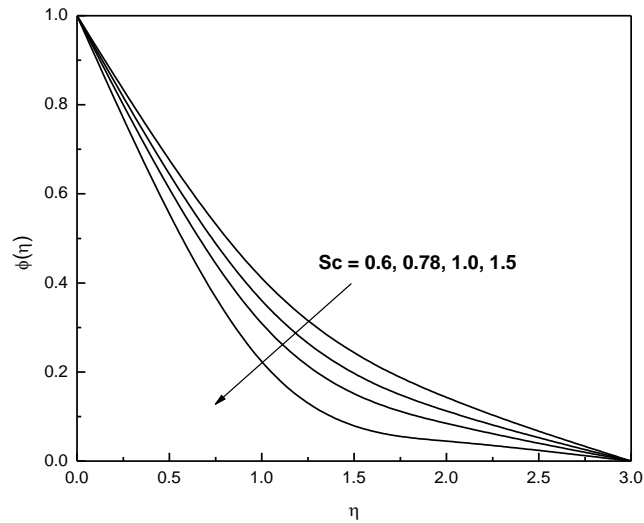


Fig. 9. Concentration profiles for varying values of Schmidt parameter (Sc)

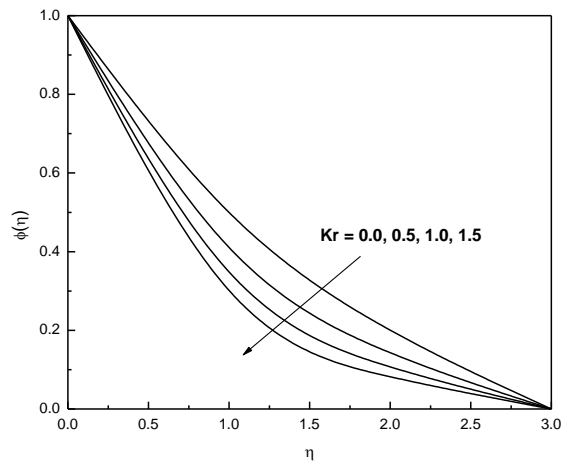


Fig. 10. Concentration profiles for varying reaction rate parameter (Kr)

Table 1: A comparison of skin-friction coefficient $-f''(0)$ for different values of M for fixed values of $Pr = R = Q = Ec = Sc = Kr = 0$.

| M | $-f''(0)$ | |
|-----|-------------------------------|---------|
| | Kameswaran <i>et al.</i> [19] | Present |
| 0.0 | 1.28181 | 1.29038 |
| 1.0 | 1.62918 | 1.63038 |
| 2.0 | 1.91262 | 1.91285 |
| 3.0 | 2.15874 | 2.15879 |
| 4.0 | 2.37937 | 2.37938 |

Table 2: Computation showing $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for different embedded flow parameter values.

| M | Pr | R | Ec | Q | Sc | Kr | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|-----|------|-----|------|-----|------|------|-----------|---------------|-------------|
| 1.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.518152 | 0.898172 |
| 2.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.6 | 0.5 | 1.91285 | 0.47744 | 0.873141 |
| 3.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.6 | 0.5 | 2.15879 | 0.445763 | 0.854063 |
| 1.0 | 1.0 | 0.5 | 0.1 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.591577 | 0.898172 |
| 1.0 | 2.0 | 0.5 | 0.1 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.832569 | 0.898172 |
| 1.0 | 0.71 | 1.0 | 0.1 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.465892 | 0.898172 |
| 1.0 | 0.71 | 2.0 | 0.1 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.417967 | 0.898172 |
| 1.0 | 0.71 | 0.5 | 0.5 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.355639 | 0.898172 |
| 1.0 | 0.71 | 0.5 | 1.0 | 0.1 | 0.6 | 0.5 | 1.63038 | 0.152498 | 0.898172 |
| 1.0 | 0.71 | 0.5 | 0.1 | 0.5 | 0.6 | 0.5 | 1.63038 | 0.354218 | 0.898172 |
| 1.0 | 0.71 | 0.5 | 0.1 | 1.0 | 0.6 | 0.5 | 1.63038 | 0.0846552 | 0.898172 |
| 1.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.78 | 0.5 | 1.63038 | 0.518152 | 1.03456 |
| 1.0 | 0.71 | 0.5 | 0.1 | 0.1 | 1.0 | 0.5 | 1.63038 | 0.518152 | 1.18749 |
| 1.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.6 | 1.0 | 1.63038 | 0.518152 | 1.06315 |
| 1.0 | 0.71 | 0.5 | 0.1 | 0.1 | 0.6 | 2.0 | 1.63038 | 0.518152 | 1.33022 |

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